Math Day 2024
at Murray State University
Upper Level Examination

- Do not open this exam until you are told to do so.
- Clearly fill in your NAME and STUDENT NUMBER on the bubble sheet. Your student number is located on the card your teacher gave you.
- You have 50 minutes to complete this exam.
- You may not use a calculator, phone, notes, book, or other aid. Any attempt to do so will result in disqualification.
- The exam will be scored as follows:

  +1 point for a correct answer

  $\frac{1}{4}$ point for an incorrect answer

  0 points for a blank answer

- Clearly select one answer on the bubble sheet for each question. If more than one answer is selected, the answer will be marked as incorrect.

GOOD LUCK!

4. Supposethere is a bag containing 2 yellow marbles, 2 red marbles, and 2 orange marbles. If two balls are drawn randomly

7. Suppose

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11. Suppose $f(x) = \sin(x)$ on the domain $\frac{}{2}; \frac{3}{2}$

17. Evaluate $\lim_{x \to 3^+} \frac{|x^2 - 7x + 12|}{x - 3}$; if the limit exists.

(a) 0

(b) 7

(c) 1

(d) 1

(e) The limit does not exist.

18. Assume you roll a 20 inch diameter ball along a line with an angular velocity of $360 \frac{\text{degrees}}{\text{second}}$. How far will the ball roll in 10 seconds?

(a) 100π inches

(b) 200π inches

(c) 400π inches

(d) 1000π inches

(e) None of the above

19. Three men are told to stand in a straight line, one in front of the other. A hat is put on each of their heads. They are told that each of these hats was selected from a group of five hats: two black hats and three white hats. The first man, standing at the front of the line, can't see either of the men behind him or their hats. The second man, in the middle, can see only the first man and his hat. The last man, at the rear, can see both other men and their hats. The last man and middle man are asked in succession if they can deduce the color of his own hat to which both cannot. What must be true?

(a) The first man has enough information to know that his hat is black.

(b) The first man has enough information to know that his hat is white.

(c) The first man would know the color of his hat only if he knew the color of the last man's hat.

(d) The first man would know the color of his hat only if he knew the color of the middle man's hat.

(e) The first man does not have sufficient information to know the color of his hat.

20. Characterize the end behavior for $f(t) = (2t + 5)^{100}(t^2 + t + 7)^{13}$.

(a) As $t \to \infty$, $f(t) \to \infty$ and as $t \to -\infty$, $f(t) \to \infty$.

(b) As $t \to \infty$, $f(t) \to \infty$ and as $t \to -\infty$, $f(t) \to -\infty$.

(c) As $t \to \infty$, $f(t) \to -\infty$ and as $t \to -\infty$, $f(t) \to \infty$.

(d) As $t \to \infty$, $f(t) \to -\infty$ and as $t \to -\infty$, $f(t) \to -\infty$.

(e) None of the above

21. Suppose a property is defined as follows:

For all x, there exists y such that $z < x$ implies $f(z) < y$.

What does it mean for this property NOT to hold?

(a) There exists x such that for all y and some $z < x$, we have $f(z) \quad y$.

(b) There exists x such that for all y and some $z \quad x$, we have $f(z) \quad y$.

(c) There exists x such that for all y, if $z < x$ then $f(z) \quad y$.

(d) There exists x such that for all y, if $z \quad x$ then $f(z) \quad y$.

(e) None of the above

22. Let n be a natural number and define $g(t) = \begin{cases} t^n; & \text{if } t > 0 \\ t^n; & \text{if } t \quad 0. \end{cases}$

If $f(x) = \dfrac{d^n}{dx^n} g(x)$, calculate $f(0)$.

(a) $f(0) = n!$

(b) $f(0) = n!$

(c) $f(0) = 0$

(d) $f(0)$ does not exist.

(e) None of the above

23. Consider the following data set:

$$1; 3; 3; 4; 5; 7; 7; 8; 8; 10; 12; 14; 20; 22; 45; 50; 60$$

What is the smallest datum point that lies in the 75th percentile?

(a) 20
(b) 22
(c) 45
(d) 50
(e) None of the above

24. How many solutions (in radians) exist for $2\pi \sin^2(x) = \pi \quad x$ in $0 \quad x \quad 2\pi$?

(a) 1
(b) 2
(c) 3
(d) 4
(e) None of the above

25. What is an equivalent to the above<<BDC /T1_2Tf <0003>Tj /T1_02l114 0 Td (pressutij /T1_0 1 Tf 1.6

65. x

27. Suppose we have the relationship $y(x)^3 + y(x) = x$. Find the equation of the tangent line to $y(x)$ at the point $(2; 1)$.

(a) $y = 1$

(b) $y = \frac{1}{6}x + \frac{2}{3}$

(c) $y = 2x + 5$

(d) $y = \frac{1}{13}x + \frac{11}{13}$

(e) None of the above

28. How many x-intercepts does the function $R(x) = \frac{x^3 + 7x^2 + 2x - 40}{x^2 - 5x + 6}$ have?

(a) 1

(b) 2

(c) 3

(d) 4

(e) 5

29. Given that $a > 0$, $\sec\theta = \frac{\sqrt{a^2 + 1}}{a}$ and that θ satisfies $\pi < \theta < \frac{3\pi}{2}$, evaluate $\tan\theta$.